Improved Procedure for CALCULATING STREAM DISCHARGE

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INTRODUCTION

Stream discharge (the volume of water passing a point in a stream during a specified interval of time) is frequently calculated from a hydrograph which relates head (i. e., water depth) to time. At any instant of time, the rate of flow can be computed by a formula which for many weirs takes the form

$$q = aH^b \quad [1]$$

where q = cubic feet per second

H = head, or depth of water (feet) in the weir

a and b = constants depending on characteristics of the weir.

With q thus available for each instant of time, the problem is to calculate discharge for any interval of time.

In the interest of simplicity, discharge is normally calculated by one of several approximate procedures.[1/] The precise procedure to be developed here is also reasonably simple, and the calculations can be performed on desk calculators without excessive difficulty. If electronic data processing machines are available, the precise procedure is certainly the recommended one.

CALCULATION PROCEDURE

Step 1

Mark the hydrograph so that the relation of head (H) to time (t) is linear between consecutive marks.

[1/] Johnson, Edward A., and Dils, Robert T. Outline for compiling precipitation, runoff, and groundwater data from small watersheds. U.S. Forest Serv. Southeast. Forest Expt. Sta. Sta. Paper 18, 40 pp., illus. 1956.

Step 2

For each linear segment on the hydrograph, tally beginning time (t_1) and corresponding head (H_1), and ending time (t_2) and head (H_2).

Step 3

Calculate discharge (Q) for each linear segment with the following formulas:

(a) when $H_2 \neq H_1$

$$Q = \frac{a}{b+1} \frac{t_2 - t_1}{H_2 - H_1} \left(H_2^{\,b+1} - H_1^{\,b+1} \right) (3600) \text{ ------ } [2]$$

(b) when $H_2 = H_1$

$$Q = aH^b (t_2 - t_1) (3600) \text{ ------------------------ } [3]$$

where Q = cubic feet and 3600 = number of seconds in an hour.

Example

Water is flowing through a right-angled V-notch weir characterized by a discharge formula

$$q = 2.52H^{2.47}$$

Preliminary analysis of the hydrograph established a linear segment between 9 a.m. and 9:30 a.m., at which times the corresponding heads were 0.25 and 0.30 foot. Thus:

$a = 2.52$	$t_1 = 9.0$	$H_1 = 0.25$
$b = 2.47$	$t_2 = 9.5$	$H_2 = 0.30$

From equation 2,

$$Q = \frac{2.52}{3.47} \frac{(9.5-9.0)}{(0.30-0.25)} (0.30^{3.47} - 0.25^{3.47}) (3600)$$

$$= 188 \text{ cubic feet.}$$

In practice, the calculations would not be as formidable as indicated above. For a given weir, the ratio $\frac{a(3600)}{b+1}$ remains a constant, and the values of $H^{3.47}$ would be obtained from tables rather than calculated individually.

Development of Equation 2

The basic integral leading to equation 2 is

$$Q = \int_{t_1}^{t_2} aH_i^b \, dt \text{ ------------------------- } [4]$$

If this integral is restricted in application to a linear segment of the head-time hydrograph, so that

$$H_i = H_1 + K(t_i - t_1) \text{ ------------------------- } [5]$$

$$\text{where } K = \frac{H_2 - H_1}{t_2 - t_1}$$

Then equation 4 may be written

$$Q = a \int_{t_1}^{t_2} \left[H_1 + K(t_i - t_1)\right]^b dt \text{ ----------------- } [6]$$

and evaluated as

$$Q = \frac{a}{b+1}\,\frac{t_2-t_1}{H_2-H_1}\left[\left[H_1 + \left(\frac{H_2-H_1}{t_2-t_1}\right)(t_2-t_1)\right]^{b+1} - \left[H_1 + \left(\frac{H_2-H_1}{t_2-t_1}\right)(t_1-t_1)\right]^{b+1}\right]$$

$$= \left(\frac{a}{b+1}\right)\left(\frac{t_2-t_1}{H_2-H_1}\right)\left(H_2^{\,b+1} - H_1^{\,b+1}\right) \text{------} [7]$$

If H is expressed in feet and t in hours, then equation 7 must be multiplied by 3600 (the number of seconds in an hour) to obtain the total discharge in cubic feet.

COMPARISON OF METHODS

Table 1 shows that discharge as calculated by commonly accepted methods may yield values considerably greater than those calculated by the integration method. For this illustration, the average rate method was used, in which

$$Q = \left[\frac{aH_1^{\,b} + aH_2^{\,b}}{2}\right](t_2-t_1)$$

and the calculations are for a right-angled V-notch weir with a rating formula $q = 2.52H^{2.47}$.

Figure 1 illustrates the fact, for a linear segment of the hydrograph, that discharge by the integration method is always less than the value obtained by other methods. During period AB, discharge rate q varies along curve EFC. The integration method evaluates the area under curve EFC. In contrast, the average-rate method averages discharge rates at times A and B, and perforce evaluates the area under line EDC.

Table 1.--Comparison of discharge by the integration and average-rate methods

Variable	Unit	Example A	Example B	Example C	Example D
Q_a 1/	Cu. ft.	946	973	1,290	1,756
Q_b 2/	Cu. ft.	946	972	1,272	1,677
Q_a-Q_b	Cu. ft.	0	1	18	79
		Basis for computations			
H_1	Foot	0.400	0.40	0.40	0.40
H_2	Foot	0.401	0.41	0.50	0.60
t_2-t_1	Hours	1.00	1.00	1.00	1.00
dH/dt	Ft./hrs.	0.001	0.01	0.10	0.20

1/ Q_a: Discharge by the average-rate method.

2/ Q_b: Discharge by the integration method.

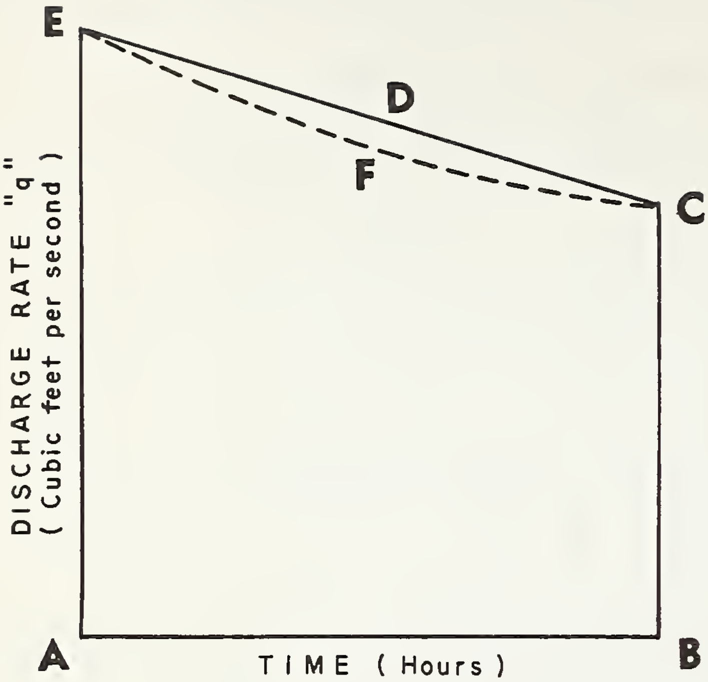

Figure 1.--Representation of stream discharge as calculated by commonly accepted methods (area ABCDE) and by the integration method (area ABCFE).

Discharge as evaluated by approximate procedures will approach the correct value if the time span between hydrograph readings is shortened so as to reduce to a minimum the difference in head. Example A, table 1, shows that with a difference in head of only 0.001 foot, the two computational methods yield the same discharge figures. The advantage of the integration method is that any time span may be utilized as long as the rate of change in head remains uniform. Thus, the true discharge will be obtained from a smaller number of hydrograph readings and with considerably reduced labor.

Bethlahmy, Nedavia
1964. Improved procedure for calculating stream discharge. U.S. Forest Serv. Res. Paper PNW-10, 6 pp., illus.

An integration method for computing discharge more accurately with fewer hydrograph readings.

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